STATIONARY WAVES

FORMATION OF STATIONARY WAVES ON STRINGS

When two identical progressives waves both travelling along the same path in opposite directions, interfere with each other, by superposition of waves resultant obtained in the form of loops, is called stationary waves.

Consider two simple harmonic progressive waves of equal amplitude 'a' propagating on a long uniform string in opposite directions each with frequency 'n' and wavelength λ.

The wave along positive X is $Y1 = a \sin 2\pi(nt - x/\lambda)$

The wave along negative X is $Y2 = a \sin 2\pi(nt + x/\lambda)$

The resultant stationary wave by principle of superposition is

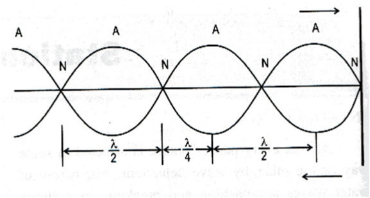
$Y = Y1 + Y2 = a \sin 2\pi(nt + x/\lambda) + a \sin 2\pi(nt - x/\lambda)$

$Y = 2a \sin(2\pi nt) \cos(2\pi x/\lambda) = A \sin(2\pi nt)$ where $A = 2a \cos(2\pi x/\lambda)$

Thus, $Y = A \sin(\omega t)$ where $A = 2a \cos(2\pi x/\lambda)$

Thus, A is the amplitude of the stationary wave and is periodic in space.

The frequency of the stationary wave is same as that of the individual progressive waves but the amplitude varies with respect to the position 'x' of the particle.



Condition for antinode

The points of the medium which vibrate with maximum amplitude are called antinode.

$A = \pm 2a$, when $\cos(2\pi x/\lambda) = \pm 1$

Hence, $2\pi x/\lambda = 0, \pi, 2\pi, \ldots\ldots.$

$x = 0, \lambda/2, \lambda, 3\lambda/2, \ldots\ldots$ is where the antinodes are produced

Distance between two antinodes is $\lambda/2$

Condition for node

The points of the medium which vibrate with minimum amplitude are called node.

$A = 0$, when $\cos(2\pi x/\lambda) = 0$

Hence, $2\pi x/\lambda = \pi/2, 3\pi/2, \ldots\ldots.$

$x = \lambda/4, 3\lambda/4, 5\lambda/4 \ldots\ldots$ is where the nodes are produced

Distance between two nodes is $\lambda/2$

The distance between node and antinode is $\lambda/4$

NOTE: The wave has alternate and evenly spaced antinode and nodes starting with antinode.

PROPERTIES OF STATIONARY WAVES

>> Two identical waves travelling in a medium in opposite directions, interfere and produce a resultant wave in the form of loops called stationary wave

>> There are some points where the displacement of the medium particles is always zero called ***nodes*** and points where the displacement of the medium particles is maximum, called ***antinode***

>> Distance between two successive antinodes (or nodes) is **λ/2**

>> Antinodes and nodes are alternately formed and the distance between an antinode and node is **λ/4**

>> The medium particles (except the node) perform SHM of the **same period** as that of the component waves but with **different amplitudes**

>> All particles with **a loop** are in the **same phase** of vibration

>> The particles in the **adjacent loop** are vibrating **out of phase**

>> Stationary wave is **periodic** in space and time

>> **Resultant wave velocity is zero** and there is **no transfer of energy**

>> *In case of longitudinal stationary waves, points of minimum displacement but maximum variation in pressure is called **pressure antinodes** and points with maximum displacement but pressure is constant (minimum change in pressure) are called **pressure nodes**

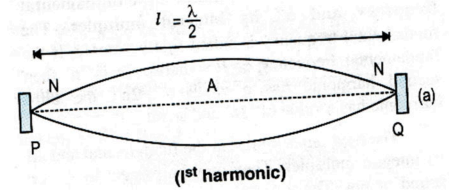
*Harmonic is used to indicate the fundamental frequency and all its integral multiples.

*Velocity of transverse wave on a string is given by $\sqrt{\frac{T}{m}}$,

where m=linear density and T= tension in the string

MODES OF VIBRATING STRINGS

Fundamental Mode/1st Harmonic/0th overtone

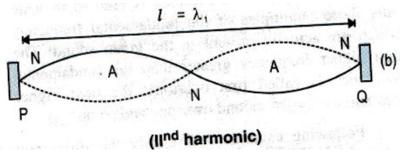


$$n = \frac{V}{\lambda} = \frac{V}{2l}$$

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

This is the lowest frequency called the fundamental frequency (1st Harmonic)

1st Overtone/2nd Harmonic

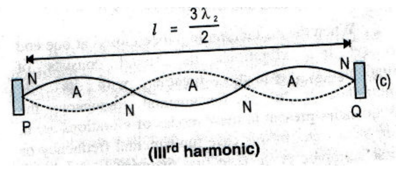


$$n_1 = \frac{V}{\lambda_1} = \frac{V}{l}$$

$$n_1 = \frac{1}{l}\sqrt{\frac{T}{m}} = 2n$$

This is the second harmonics (since frequency is twice the fundamental)

2nd overtone/3rd Harmonic



$$n_2 = \frac{V}{\lambda_2} = \frac{3V}{2l}$$

$$n_2 = \frac{3}{2l}\sqrt{\frac{T}{m}} = 3n$$

This is the third harmonics (since frequency is thrice the fundamental)

In General, pth overtone is (p+1)th harmonic means (p+1)n as the frequency.

*Thus a stretched string produces all harmonics

LAWS OF VIBRATING STRINGS

(1) ***Law of Length***: The fundamental frequency of transverse vibration of a stretched string is inversely proportional to the vibrating length if the tension (T) in the string and linear density (m) of the string are kept constant.

$n \propto 1/l$ (if T and m are constant) , $n_1 l_1 = n_2 l_2 = $constant

In general , $n\,l$ =constant

(2) ***Law of Tension***: The fundamental frequency of transverse vibration of a stretched string is directly proportional to the square root of the tension (T) in the string if linear density (m) and vibrating length(**l**) of the string are kept constant.

$n \propto \sqrt{T}$ (if **l** and m are constant)

$$\frac{n_1}{\sqrt{T_1}} = \frac{n_2}{\sqrt{T_2}} = constant$$

In general , $\frac{n}{\sqrt{T}} = constant$

(3) ***Law of Linear density(Law of mass per unit length)***: The fundamental frequency of transverse vibration of a stretched string is inversely proportional to the square root of the linear density(m) of the string if Tension (T) and vibrating length(**l**) of the string are kept constant.

$n \propto 1/\sqrt{m}$ (if **l** and T are constant) , $n_1\sqrt{m_1} = n_2\sqrt{m_2} = constant$

In general , $n\sqrt{m} = constant$

NOTE: $m = \frac{M}{L} = \frac{\rho V}{L} = \frac{\rho A L}{L} = \rho A = \rho \pi r^2$,

$$and\ since\ n\sqrt{m} = const\ thus\ n_1\sqrt{m_1} = n_2\sqrt{m_2}$$

$$n_1\sqrt{\rho\pi {r_1}^2} = n_2\sqrt{\rho\pi {r_2}^2}$$

$$assuming\ same\ wire\ \rho\ will\ be\ same, thus\ n_1 r_1 = n_2 r_2$$

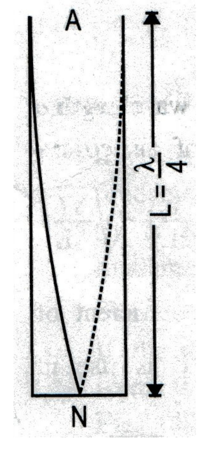
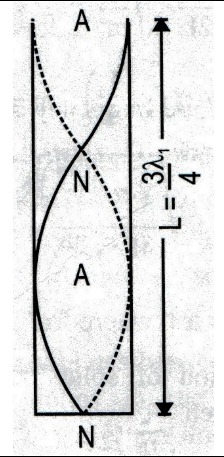
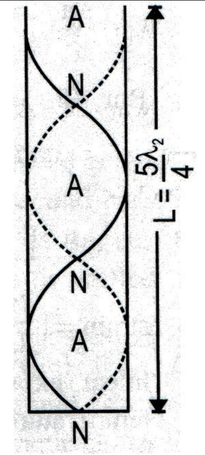
VIBRATION OF AIR COLUMN CLOSED AT ONE END

A long cylindrical closed organ pipe with one end closed and a vibrating tuning fork is held horizontally near the open end of this closed pipe.

(a) The compression produced then at the ends get reflected at the rigid boundary as a compression (because there is least freedom for motion of particles) and at the open end gets reflected as a rarefaction (because there is maximum freedom for motion of particles).

(b) The rarefaction produced by the tuning fork get reflected at the rigid boundary as a rarefaction. The amplitude of vibration of air column inside becomes maximum i.e. stationary waves are formed (it is due to resonance)

NOTE: the closed end is always a displacement node (pressure antinode) and the open end is a displacement antinode (pressure node)

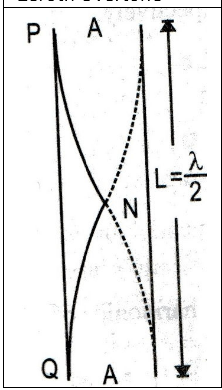
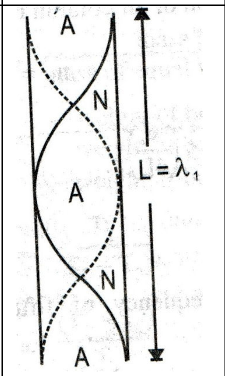
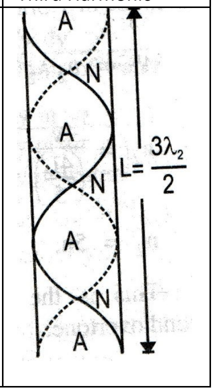
Fundamental Note First Mode Zeroth Overtone	Second Mode First Overtone Third Harmonic	Third Mode Second Overtone Fifth Harmonic
		
$\lambda=4L$ $n_0=\frac{V}{\lambda}=\frac{V}{4L}$	$\lambda_1=\frac{4L}{3}$ $n_1=\frac{V}{\lambda_1}=\frac{3V}{4L}=3n_0$	$\lambda_2=\frac{4L}{5}$ $n_2=\frac{V}{\lambda_2}=\frac{5V}{4L}=5n_0$
• Relation between frequencies is 1:3:5 • Only odd harmonics are present as overtone • p[th] overtone is (2p+1) harmonic i.e. $n=(2p+1)n_0$		

VIBRATION OF AIR COLUMN OPEN AT BOTH ENDS

A long cylindrical open organ pipe with both ends open and a vibrating tuning fork is held horizontally near one of the open ends.

(a) The compression produced by the tuning fork, travels through the air column and gets reflected as rarefaction at the other open end (there is maximum freedom of motion of air column).

(b) The rarefaction moves opposite and gets reflected as compression at the other open end.

Fundamental Note First Mode Zeroth Overtone	Second Mode First Overtone Second Harmonic	Third Mode Second Overtone Third Harmonic
		
$\lambda=2L$ $n_0=\frac{V}{\lambda}=\frac{V}{2L}$	$\lambda_1=L$ $n_1=\frac{V}{\lambda_1}=\frac{V}{L}=2n_0$	$\lambda_2=\frac{2L}{3}$ $n_2=\frac{V}{\lambda_2}=\frac{3V}{2L}=3n_0$
• Relation between frequencies is 1:2:3 • All harmonics are present as overtone • p[th] overtone is (p+1) harmonic i.e. $n=(p+1)n_0$		

END CORRECTION

In vibrating air column, boundary conditions we assumed that antinode is formed at the open end and node is formed at the closed end. The antinode is not formed exactly at the open end. Distance between the antinode and the open end is called **end correction**. It is calculated by e=0.3d, where **d is the inner diameter** of the tube.

Hence the corrected length = L+0.3d for closed at one end and L+0.6d for open at both ends.

Causes of end correction: End correction arises because air particle in the plane of the open end of the tube, are not free to move in all directions. Hence reflection takes place at the plane, at small distance outside the tube.

Limitations of end correction:
- Inner diameter of the tube must be uniform throughout
- Effect of flow of air outside tube is to be neglected
- Effect of temperature of air outside is to be neglected
- The prongs of vibrating tuning fork must be held horizontally at the centre and a small distance above the open end of the tube

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Miscellaneous:

For pipe closed at one end: $4n_1(L_1+e)=4n_2(L_2+e)$

Thus, $e=\frac{n_1l_1-n_2l_2}{n_2-n_1}$

For pipe closed at open end: $2n_1(L_1+2e)=2n_2(L_2+2e)$

Thus, $e=\frac{n_1l_1-n_2l_2}{2(n_2-n_1)}$

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